Bulk fields and supersymmetry in a slice of AdS

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Abstract

Five-dimensional models where the bulk is a slice of AdS have the virtue of solving the hierarchy problem. The electroweak scale is generated by a "warp" factor of the induced metric on the brane where the standard model fields live. However, it is not necessary to confine the standard model fields on the brane and we analyze the possibility of having the fields actually living in the slice of AdS. Specifically, we study the behaviour of fermions, gauge bosons and scalars in this geometry and their implications on electroweak physics. These scenarios can provide an explanation of the fermion mass hierarchy by warp factors. We also consider the case of supersymmetry in the bulk, and analyze the conditions on the mass spectrum. Finally, a model is proposed where the warp factor generates a small (TeV) supersymmetry-breaking scale, with the gauge interactions mediating the breaking to the scalar sector.

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1 Introduction

One the major puzzles in particle physics is the large hierarchy of scales that appear in the standard model (SM). The Planck scale, $M_P \sim 10^{18}$ GeV, is much larger than the electroweak scale (~ 100 GeV), and even larger than the electron or neutrino mass. It has recently been realized that the origin of these hierarchies in scale can be related to the presence of extra dimensions with nontrivial spacetime geometries [1, 2]. An explicit example of this is the Randall-Sundrum model [2]. This consists of a five-dimensional theory where the extra dimension is compactified on an orbifold, and the bulk geometry is a slice of anti-de Sitter (AdS$_5$) space of length πR. Hence, in this theory there are two four-dimensional boundaries. The induced metric on these boundaries differs by an exponential ("warp") factor, and generates two effective scales, M_P and $M_P e^{-\pi kR}$ (where k is the AdS curvature scale of order M_P). Thus, for a moderately large extra dimension, $kR \gtrsim 1$, one can obtain an exponentially small scale, $M_P e^{-\pi kR}$, on one of the boundaries. If the SM fields live on this boundary, then the electroweak scale can be associated with the effective scale on this boundary. In particular for $kR \sim 12$, this will be $\mathcal{O}(\text{TeV})$ and the brane is referred to as the TeV-brane [3].

In the model of Ref. [2] only gravity propagates in the 5D bulk, while the SM fields are on the TeV-brane. It is interesting to study other possibilities, such as having all the SM fields in the 5D bulk. This is the subject of this article, where we will study the behaviour of various spin fields in a slice of AdS$_5$. We will first analyze the non-supersymmetric case. Part of this analysis has been carried out previously in the literature. The role of scalar fields in the bulk was first considered in Ref. [4]. Subsequent studies considered the SM gauge bosons [5, 6], and fermions [7] in the bulk. The complete SM in the bulk was considered in Ref. [8, 9]. We will present here a more compact and general analysis, including for example, the case when scalar fields have boundary mass terms. As an important new ingredient, we will consider supersymmetry in the AdS$_5$ slice. We will derive the conditions necessary for supersymmetry and the implications for the particle spectrum.

Armed with the above, we will then study the phenomenological consequences of having the SM fields in the 5D bulk. We will show that SM fermions and gauge bosons can propagate in the AdS$_5$ bulk without conflicting with any experimental data. However, the Higgs field must arise from a Kaluza-Klein excitation that is localized by the AdS metric on the TeV-brane [4, 8]. We will show that having the SM fermions in the bulk can explain the fermion mass hierarchy by means of the metric warp factor. We will also study the magnitude of higher-dimensional operators. These operators are known to be a problem in theories with a low (TeV) cutoff scale (including in the original Randall-Sundrum setup [10]), since they can induce large proton decay rates or large flavour violating interactions. These operators can be suppressed by allowing the fermions to be off the TeV-brane. This leads to partial success because flavour violation can be suppressed to the desirable level, but this is not the case for proton decay.

The inclusion of supersymmetry allows for new alternatives. The Higgs field can now be delocalized from the TeV-brane, together with the fermions. This delocalization is possible thanks to supersymmetry that protects the Higgs massless mode far from the TeV-brane. If fermions are also distant from the TeV-brane, then proton decay can be sufficiently suppressed,

while at the same time allowing for the fermion masses to be generated. In fact, this model at low energy resembles the ordinary MSSM. Nevertheless, it does have important differences. For instance, the boundary TeV-brane can now be the source of supersymmetry breaking and a window to "Planckian" physics. Gauge superfields living in the bulk will be responsible for mediating TeV-brane physics to fields that are far away from it. For example, the gauge interactions can be the messengers of supersymmetry breaking to the quark and lepton sector, guaranteeing a universal scalar mass spectrum. Similar scenarios were considered in the case of a flat extra dimension. This was done either by breaking supersymmetry in the bulk [11] or on a distant brane [12]. In Refs. [11], a large (TeV) extra dimension was required that implied a cutoff (string) scale of $\mathcal{O}$(TeV). In Refs. [12], a small (Planckian) extra dimension was advocated, but in this case the resultant phenomenology was similar to the gravity-mediated models of supersymmetry-breaking proposed earlier with no new low-energy implications. Nevertheless, the possibility here of mediating supersymmetry-breaking by five-dimensional gauge bosons leads to a very different scenario with respect to the previous examples. As we will see, we can combine the good implications of having a large ($\sim M_P$) cutoff scale (such as small proton decay, small neutrino masses), with the new phenomenology of having an extra dimension (Kaluza-Klein states at collider physics, quantum gravity effects at the TeV).

It is important to point out why the slice of AdS_5 is interesting and different from other compactifications [13]. Theories that solve the hierarchy problem with other compactifications [1, 13] usually require a large compactification radius (or, equivalently, a large volume in the extra dimensions). Therefore, if SM fields propagate in these extra dimensions, the couplings become too weak.

Finally, three important remarks are in order. Even though we will be considering fields other than the graviton living in the 5D bulk, we will assume that the AdS metric is not modified by the presence of these bulk fields. In other words, the back-reaction from the bulk fields is neglected. Second, there have been recent unsuccessful attempts at deriving the scenario of Ref. [2] from a five-dimensional supergravity Lagrangian [14]. While there is no general proof that a description from a more fundamental theory does not exist, we will not have anything more to add here. Further studies must still be carried out to settle this issue. We will assume that the solution of Ref. [2] exists and study its compatibility with supersymmetry. Finally, to obtain the AdS slice, the TeV-brane in Ref. [2] requires a negative tension. This will violate the weak-energy condition [15]. Nevertheless, it is conceivable that a setup, like we are considering, is possible without violating the weak-energy condition [15], especially given the fact that the fundamental theory is most likely to be supersymmetric.

The article is organized in the following way. In Section 2, we introduce the compactification scenario of Ref. [2], and derive the Kaluza-Klein decomposition of fields with different spins. In Section 3, we study the conditions necessary for supersymmetry in a slice of AdS_5. In particular, we derive the mass spectrum and wavefunctions of the gauge supermultiplet and hypermultiplet. The possibility of having the SM fermions and gauge bosons in the 5D bulk is studied in Section 4, where particular attention is paid to constraints from electroweak data. This will lead us to consider supersymmetric models, together with their resulting phenomenological predictions. Our conclusions and final comments appear in Section 5.

2 Bulk fields in a slice of AdS$_5$

We will consider the scenario of Ref. [2], based on a non-factorizable geometry with one extra dimension. In this scenario, the fifth dimension y is compactified on an orbifold, $S^1/\mathbb{Z}_2$ of radius R, with $-\pi R \leq y \leq \pi R$. The orbifold fixed points at, $y^* = 0$ and $y^* = \pi R$ are also the locations of two 3-branes, which form the boundary of the five-dimensional space-time. Consequently, the classical action for this configuration is given by

$$S = -\int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-g} \left[\Lambda + \frac{1}{2} M_5^3 \mathcal{R} + \delta(y) \left(\Lambda_{(0)} + \mathcal{L}_{(0)} \right) + \delta(y - \pi R) \left(\Lambda_{(\pi R)} + \mathcal{L}_{(\pi R)} \right) \right] \, , \tag{1}$$

where M_5 is the five-dimensional Planck mass scale and $\mathcal{R}$ is the five-dimensional Ricci scalar, constructed from the five-dimensional metric g_{MN}, with the 5D coordinates labelled by capital Latin letters, $M = (\mu, 5)$. The cosmological constants in the bulk and boundary are Λ, and $\Lambda_{(y^*)}$, respectively.

A solution to the five-dimensional Einstein's equations, which respects four-dimensional Poincare invariance in the x^μ directions, is given by [2]

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \, , \tag{2}$$

where

$$\sigma = k|y| \, , \tag{3}$$

and $1/k$ is the AdS curvature radius. The four-dimensional metric is $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ with $\mu = 1, .., 4$. The metric solution (2) is valid provided that the bulk and boundary cosmological constants are related by

$$\begin{aligned} \Lambda &= -6M_5^3 k^2 \, , \\ \Lambda_{(0)} &= -\Lambda_{(\pi R)} = -\frac{\Lambda}{k} \, . \end{aligned} \tag{4}$$

The four-dimensional reduced Planck scale M_P is related to M_5 in the following way:

$$M_P^2 = \frac{M_5^3}{k} \left(1 - e^{-2\pi kR} \right) \, . \tag{5}$$

From the form of the metric solution (2), the space-time between the two branes located at $y^* = 0$ and $y^* = \pi R$ is simply a slice of AdS$_5$ geometry. The effective mass scale on the brane at $y^* = 0$ is the Planck scale M_P, and we will refer to this 3-brane as the Planck-brane. Similarly, at $y^* = \pi R$, the effective mass scale is $M_P e^{-\pi kR}$, which will be associated with the TeV scale provided $kR \simeq 12$. This 3-brane will be referred to as the TeV-brane. It is also interesting to note that the relations (4) arise in the effective five-dimensional Horava-Witten theory [16] when the Calabi-Yau moduli are fixed.

So far we have only assumed gravity to be present in the five-dimensional bulk. We will now study a U(1) gauge field, V_M, a complex scalar, ϕ, and a Dirac fermion, Ψ, living in a slice of AdS$_5$ given by the metric Eq. (2). The five-dimensional (5D) bulk action, S_5, has kinetic energy and mass terms given by

$$S_5 = -\int d^4x \int dy \sqrt{-g} \left[\frac{1}{4g_5^2} F_{MN}^2 + |\partial_M \phi|^2 + i\bar{\Psi}\gamma^M D_M \Psi + m_\phi^2 |\phi|^2 + im_\Psi \bar{\Psi}\Psi \right], \quad (6)$$

where $g = \det(g_{MN})$. The U(1) gauge field strength is $F_{MN} = \partial_M V_N - \partial_N V_M$ and in curved space the covariant derivative is $D_M = \partial_M + \Gamma_M$, where Γ_M is the spin connection. In particular, for the metric defined by Eq. (2), we have

$$\Gamma_\mu = \frac{1}{2}\gamma_5\gamma_\mu \frac{d\sigma}{dy} \quad \text{and} \quad \Gamma_5 = 0. \quad (7)$$

The gamma matrices, $\gamma_M = (\gamma_\mu, \gamma_5)$ are defined in curved space as $\gamma_M = e_M^\alpha \gamma_\alpha$, where e_M^α is the vierbein and γ_α are the Dirac matrices in flat space.

Under the $\mathbb{Z}_2$ parity the boson fields can be defined either odd or even (depending on their interactions). For the fermion Ψ, the $\mathbb{Z}_2$ transformation is given by $\Psi(-y) = \pm\gamma_5\Psi(y)$, where the arbitrariness of the sign can only be determined by the fermion interactions. We then have that $\bar{\Psi}\Psi$ is odd under the $\mathbb{Z}_2$ symmetry, and consequently the Dirac mass parameter m_Ψ must also be odd. This nontrivial transformation of the fermion mass parameter implies that m_Ψ must arise from an underlying scalar field that receives a vacuum expectation value (VEV) with an odd 'kink' profile. In other words, the vacuum configuration of the scalar field can be thought of as an infinitely thin domain wall. This is very similar to the background 3-form field in the dimensional reduction of Horava-Witten theory [16]. Furthermore, note that a Dirac mass term cannot be induced on the boundaries, since $\bar{\Psi}\Psi$ vanishes there. On the other hand, a Majorana mass term for the fermion can be added, either in the bulk or on the boundaries. This term will necessarily break supersymmetry, and we will comment on this in section 4. The scalar mass term is even under the $\mathbb{Z}_2$ symmetry and can be either a bulk or boundary term. Therefore, the mass parameters of the scalar and fermion fields can be parametrized as [1]

$$\begin{aligned} m_\phi^2 &= ak^2 + b\sigma'', \\ m_\Psi &= c\sigma', \end{aligned} \quad (8)$$

where a, b and c are arbitrary dimensionless parameters, and the derivatives are defined as

$$\begin{aligned} \sigma' &= \frac{d\sigma}{dy} = k\epsilon(y), \\ \sigma'' &= \frac{d^2\sigma}{dy^2} = 2k\left[\delta(y) - \delta(y - \pi R)\right]. \end{aligned} \quad (9)$$

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[1]We are assuming that the magnitude of the boundary mass for the scalar is the same on the two boundaries, but with opposite sign. As we will see, this is required by supersymmetry. The generalization to different masses for different boundaries can be easily obtained from the analysis here.

The step function $\epsilon(y)$ is defined as being 1 (-1) for positive (negative) y, and $\delta(y)$ is the Dirac delta-function.

The equations of motion for the gauge, scalar and fermion fields are respectively given by

$$\partial_M(\sqrt{-g}g^{MN}g^{RS}F_{NS}) = 0,$$
$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\phi) - m_\phi^2\phi = 0,$$
$$(g^{MN}\gamma_M D_N + m_\Psi)\Psi = 0. \tag{10}$$

Using the metric of Eq. (2), one can write a general second-order differential equation [2]

$$\left[e^{2\sigma}\eta^{\mu\nu}\partial_\mu\partial_\nu + e^{s\sigma}\partial_5(e^{-s\sigma}\partial_5) - M_\Phi^2\right]\Phi(x^\mu, y) = 0, \tag{11}$$

where $\Phi = \{V_\mu, \phi, e^{-2\sigma}\Psi_{L,R}\}$, $s = \{2, 4, 1\}$ and $M_\Phi^2 = \{0, ak^2 + b\sigma'', c(c \pm 1)k^2 \mp c\sigma''\}$. For the fermion, we have introduced the exponential factor, $e^{-2\sigma}$, which takes into account the spin connection, and we have explicitly separated the left and right components (defined by $\Psi_{L,R} = \pm\gamma_5\Psi_{L,R}$).

2.1 Kaluza-Klein decomposition

Let us decompose the 5D fields as

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=0}^{\infty}\Phi^{(n)}(x^\mu)f_n(y), \tag{12}$$

where the Kaluza-Klein modes $f_n(y)$ obey the orthonormal condition

$$\frac{1}{2\pi R}\int_{-\pi R}^{\pi R}dy\, e^{(2-s)\sigma}f_n(y)f_m(y) = \delta_{nm}. \tag{13}$$

Thus from Eq. (11), the Kaluza-Klein eigenmodes $f_n(y)$ satisfy the differential equation

$$\left[-e^{s\sigma}\partial_5(e^{-s\sigma}\partial_5) + \widehat{M_\Phi^2}\right]f_n = e^{2\sigma}m_n^2 f_n. \tag{14}$$

In Eq. (14), we have defined the mass-squared parameter, $\widehat{M_\Phi^2} = \{0, ak^2, c(c\pm1)k^2\}$, without the boundary mass terms proportional to σ''. These terms will be considered later when we impose the boundary conditions on the solutions. The corresponding eigenfunctions f_n, obtained by solving Eq. (14), are

$$f_n(y) = \frac{e^{s\sigma/2}}{N_n}\left[J_\alpha(\frac{m_n}{k}e^\sigma) + b_\alpha(m_n)\,Y_\alpha(\frac{m_n}{k}e^\sigma)\right], \tag{15}$$

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[2]For the gauge field we work, as in the graviton case [2], in the gauge where $V_5 = 0$ together with the constraint $\partial_\mu V^\mu = 0$.

where m_n is the mass of the Kaluza-Klein excitation $\Phi^{(n)}$ and the normalization factor N_n and coefficients $b_\alpha(m_n)$ are constants. The Bessel functions J_α and Y_α are of order $\alpha = \sqrt{(s/2)^2 + \widehat{M}_\Phi^2/k^2}$. Using the orthonormal condition Eq. (13), an expression for the normalization constant N_n is

$$N_n^2 = \frac{1}{\pi R} \int_0^{\pi R} dy \, e^{2\sigma} \left[J_\alpha(\frac{m_n}{k}e^\sigma) + b_\alpha(m_n)Y_\alpha(\frac{m_n}{k}e^\sigma) \right]^2 . \tag{16}$$

In the limit $m_n \ll k$ and $kR \gg 1$ an approximate expression for the normalization constant is

$$N_n \simeq \frac{e^{\pi kR}}{\sqrt{2\pi kR}} J_\alpha(\frac{m_n}{k}e^{\pi kR}) \simeq \frac{e^{\pi kR/2}}{\sqrt{\pi^2 Rm_n}} . \tag{17}$$

Note that in this limit the normalization constant, N_n, does not depend on the particular type of field i.e. the parameter s. The Kaluza-Klein masses m_n and the coefficients $b_\alpha(m_n)$ are determined by imposing the boundary conditions on the solution (15). The appropriate boundary condition depends on whether the field is odd or even under the orbifold $\mathbb{Z}_2$ symmetry.

Even fields:

If the 5D field is even under the $\mathbb{Z}_2$ symmetry, then we have

$$f_n(y) \rightarrow f_n(|y|) . \tag{18}$$

The derivative of f_n must be either discontinuous (proportional to $\epsilon(y)$) or continuous on the boundaries depending on whether or not the mass-squared parameter, M_Φ^2 in Eq. (11), has delta function terms, σ''. Thus we must impose

$$\left(\frac{df_n}{dy} - r\sigma' f_n \right) \bigg|_{0,\pi R} = 0 , \tag{19}$$

where the parameter r has the values $r = \{0, b, \mp c\}$ for $\Phi = \{V_\mu, \phi, e^{-2\sigma}\Psi_{L,R}\}$, respectively. Imposing Eq. (19) gives rise to the two equations

$$b_\alpha(m_n) = -\frac{(-r + s/2)J_\alpha(\frac{m_n}{k}) + \frac{m_n}{k}J_\alpha'(\frac{m_n}{k})}{(-r + s/2)Y_\alpha(\frac{m_n}{k}) + \frac{m_n}{k}Y_\alpha'(\frac{m_n}{k})} , \tag{20}$$

$$b_\alpha(m_n) = b_\alpha(m_n e^{\pi kR}) . \tag{21}$$

These two conditions determine the values of b_α and m_n. In the limit that $m_n \ll k$ and $kR \gg 1$ the Kaluza-Klein mass solutions for $n = 1, 2, \ldots$ and $\alpha > 0$ are

$$m_n \simeq (n + \frac{\alpha}{2} - \frac{3}{4})\pi k e^{-\pi kR} . \tag{22}$$

The approximate mass formula (22) becomes more exact, the larger the value of n.

Odd fields:

If the 5D field is odd under the $\mathbb{Z}_2$ symmetry, then we have

$$f_n(y) \to \frac{\sigma'}{k} f_n(|y|) \,. \tag{23}$$

The continuity of f_n at the boundaries implies that

$$f_n\Big|_{0,\pi R} = 0 \,, \tag{24}$$

and consequently

$$b_\alpha(m_n) = -\frac{J_\alpha\left(\frac{m_n}{k}\right)}{Y_\alpha\left(\frac{m_n}{k}\right)} \,, \tag{25}$$

$$b_\alpha(m_n) = b_\alpha(m_n e^{\pi k R}) \,. \tag{26}$$

In this case one can check that the derivative of f_n is continuous on the boundaries and does not lead to further conditions. As in the even case, an approximate solution for the Kaluza-Klein tower in the limit that $m_n \ll k$ and $kR \gg 1$ is

$$m_n \simeq (n + \frac{\alpha}{2} - \frac{1}{4})\pi k e^{-\pi k R} \,, \tag{27}$$

where $n = 1, 2, \ldots$ and $\alpha > 0$. Note that the difference in the approximate mass formulae between the even and odd mass solutions is $\pi/2$.

In the case of fermion fields one should note that the even and odd functions are related by coupled first-order differential equations

$$\gamma^\mu \partial_\mu \hat{\Psi}_R + \partial_5 \hat{\Psi}_L + m_\Psi \hat{\Psi}_L = 0 \,, \tag{28}$$

$$\gamma^\mu \partial_\mu \hat{\Psi}_L - \partial_5 \hat{\Psi}_R + m_\Psi \hat{\Psi}_R = 0 \,, \tag{29}$$

where $\hat{\Psi} = e^{-2\sigma}\Psi$. Thus, the even and odd fermion fields must also satisfy these equations.

3 Supersymmetry in a slice of AdS$_5$

An AdS space is compatible with supersymmetry [17]. However, in contrast to the flat space case, AdS supersymmetry requires that different fields belonging to the same supersymmetric multiplet have different masses. This is because the momentum operator, P, in AdS space does not commute with the supersymmetric charges and P^2 is not a Casimir operator.

The supermultiplet mass-spectrum has been derived in Ref. [18] for a five-dimensional AdS space. Here we will extend the analysis to the case where the fifth dimension is compactified on an orbifold $S^1/\mathbb{Z}_2$. In the next section, we will see that the supermultiplet mass-spectrum in a slice of AdS$_5$ will have interesting phenomenological implications.

3.1 Supergravity multiplet

The on-shell supergravity multiplet consists of the vierbein e_M^α, the graviphoton B_M and two symplectic-Majorana [3] gravitinos Ψ_M^i ($i = 1, 2$). The index i labels the fundamental representation of the SU(2) automorphism group of the $N = 1$ supersymmetry algebra in five dimensions. In a slice of AdS$_5$, the supergravity Lagrangian has extra terms proportional to the cosmological constants:

$$S_5 = -\frac{1}{2} \int d^4x \int dy \sqrt{-g} \left[M_5^3 \left\{ \mathcal{R} + i\bar{\Psi}_M^i \gamma^{MNR} D_N \Psi_R^i - i\frac{3}{2}\sigma' \bar{\Psi}_M^i \sigma^{MN} (\sigma_3)^{ij} \Psi_N^j \right\} + 2\Lambda - \frac{\Lambda}{k^2}\sigma'' \right], \tag{30}$$

where $\gamma^{MNR} \equiv \sum_{\text{perm}} (-1)^p \gamma^M \gamma^N \gamma^R / 3!$ and $\sigma^{MN} = [\gamma^M, \gamma^N]/2$. In Eq. (30) we do not show the dependence on B_M, since in the AdS$_5$ background we set $B_M = 0$. In order to respect supersymmetry in AdS$_5$, the supersymmetric transformation of the gravitino must be changed, with respect to the supergravity case with no cosmological constant, in the following way

$$\delta\Psi_M^i = D_M \eta^i + \frac{\sigma'}{2}\gamma_M (\sigma_3)^{ij} \eta^j \,, \tag{31}$$

where $\sigma_3 = \text{diag}(1, -1)$ and the symplectic-Majorana spinor η^i is the supersymmetric parameter. Without loss of generality, we have defined the $\mathbb{Z}_2$ transformation of the symplectic-Majorana spinor as

$$\eta^i(-y) = (\sigma_3)^{ij} \gamma_5 \eta^j(y) \,. \tag{32}$$

The condition that the AdS$_5$ background does not break supersymmetry is $\delta\Psi_M^i = 0$, and using Eq. (31) this leads to the Killing spinor equation

$$D_M \eta^i = -\frac{\sigma'}{2}\gamma_M (\sigma_3)^{ij} \eta^j \,. \tag{33}$$

In a non-compact five-dimensional AdS space this condition is always fulfilled. However in the orbifold compactification, the boundary terms require an extra condition to be satisfied, namely

$$\gamma_5 \eta^i = (\sigma_3)^{ij} \eta^j \,. \tag{34}$$

This condition implies that only half of the 5D supersymmetric charges are preserved. Therefore after compactification, one has in 4D a $N = 1$ supersymmetric theory instead of $N = 2$.

3.2 Vector supermultiplet

The on-shell field content of the vector supermultiplet is $\mathbb{V} = (V_M, \lambda^i, \Sigma)$ where V_M is the gauge field, λ^i is a symplectic-Majorana spinor, and Σ is a real scalar in the adjoint representation.

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[3] We will follow the conventions of Ref. [19] (except for our metric convention $\eta_{\mu\nu} = (-1, 1, 1, 1)$) where the two symplectic-Majorana spinors, ψ^i and χ^i, satisfy $\bar{\psi}^i \gamma^M \ldots \gamma^P \chi^j = -\epsilon^{ik} \epsilon^{jl} \bar{\chi}^l \gamma^P \ldots \gamma^M \psi^k$.

For simplicity we will consider a U(1) gauge group. The action has the form

$$S_5 = -\frac{1}{2} \int d^4x \int dy \sqrt{-g} \left[\frac{1}{2g_5^2} F_{MN}^2 + (\partial_M \Sigma)^2 + i\bar{\lambda}^i \gamma^M D_M \lambda^i + m_\Sigma^2 \Sigma^2 + im_\lambda \bar{\lambda}^i (\sigma_3)^{ij} \lambda^j \right].$$
(35)

In flat space, supersymmetry requires that $m_\Sigma = m_\lambda = 0$. This is to be contrasted with an AdS$_5$ background, where fields in the same supermultiplet must have different masses [18]. By requiring Eq. (35) to be invariant under the supersymmetric transformations

$$\begin{aligned}
\delta V_M &= -i\bar{\eta}^i \gamma_M \lambda^i , \\
\delta \Sigma &= \bar{\eta}^i \lambda^i , \\
\delta \lambda^i &= (-\sigma^{MN} F_{MN} + i\gamma^M \partial_M \Sigma)\eta^i - 2i\sigma' \Sigma (\sigma_3)^{ij} \eta^j ,
\end{aligned}$$
(36)

where η^i satisfies the Killing equation (33) and the condition (34), one finds that in a slice of AdS$_5$, the five-dimensional masses of the scalar and spinor fields in the vector supermultiplet must be

$$\begin{aligned}
m_\Sigma^2 &= -4k^2 + 2\sigma'' , \\
m_\lambda &= \frac{1}{2}\sigma' .
\end{aligned}$$
(37)

The Kaluza-Klein decomposition can be obtained easily from the analysis of the previous sections. From Eq. (37) and Eq. (8) we have

$$a = -4 , \quad b = 2 , \quad \text{and} \quad c = \frac{1}{2} .$$
(38)

Using Eq. (38), we find that $\alpha = 1$ for V_μ and λ_L^1, while $\alpha = 0$ for Σ and λ_L^2. If we assume that V_μ and λ_L^1 are even, while Σ and λ_L^2 are odd, then the Kaluza-Klein masses are determined by the equation

$$\frac{J_0(\frac{m_n}{k})}{Y_0(\frac{m_n}{k})} = \frac{J_0(\frac{m_n}{k}e^{\pi kR})}{Y_0(\frac{m_n}{k}e^{\pi kR})} .$$
(39)

Thus, even though the values of α are different, we find as expected that all fields of the supermultiplet have identical Kaluza-Klein masses. In fact, the approximate solution for the mass of the Kaluza-Klein modes with $n = 1, 2, \ldots$ is given by [6]

$$m_n \simeq (n - \frac{1}{4})\pi k e^{-\pi kR} ,$$
(40)

and is consistent with the mass formulae (22) and (27). The even fields V_μ and λ_L^1 will have a massless mode with the following y dependence:

$$\begin{aligned}
V_\mu(x,y) &= \frac{1}{\sqrt{2\pi R}} V_\mu^{(0)}(x) + \ldots , \\
\lambda_L^1(x,y) &= \frac{e^{3\sigma/2}}{\sqrt{2\pi R}} \lambda_L^{1\,(0)}(x) + \ldots .
\end{aligned}$$
(41)

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Notice that the zero modes have the proper y dependence to make the 4D kinetic terms of these modes in the action

$$
-\int d^4x \frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy \sqrt{-g} \left[\frac{1}{2g_5^2} \partial^\nu V^{\mu\,(0)}(x) \partial_\nu V_\mu^{(0)}(x) + i e^{3\sigma} \bar{\lambda}_L^{1\,(0)}(x) \gamma^\mu \partial_\mu \lambda_L^{1\,(0)}(x) \right] \,, \qquad (42)
$$

invariant under the conformal transformation $g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$. This implies that the action (42) in the background metric (2) is independent of the y coordinate. Consequently, the zero modes are not localized by the AdS space-time and they behave like zero modes in flat space. Furthermore, their couplings to the two boundaries are of equal strength.

The odd fields Σ and λ_L^2 do not have massless modes because this is not consistent with the orbifold condition. Therefore, the massless sector from V_μ and λ_L^1 forms an $N = 1$ supersymmetric vector multiplet.

3.3 Hypermultiplet

The hypermultiplet consists of $\mathbb{H} = (H^i, \Psi)$ where H^i are two complex scalars and Ψ is a Dirac fermion. The action has the form

$$
S_5 = -\int d^4x \int dy \sqrt{-g} \left[\left| \partial_M H^i \right|^2 + i \bar{\Psi} \gamma^M D_M \Psi + m_{H^i}^2 |H^i|^2 + i m_\Psi \bar{\Psi} \Psi \right] . \qquad (43)
$$

Invariance under the supersymmetric transformations

$$
\begin{aligned}
\delta H^i &= \sqrt{2} i \epsilon^{ij} \bar{\eta}^j \Psi \,, \\
\delta \Psi &= \sqrt{2} \left[\gamma^M \partial_M H^i \epsilon^{ij} - \frac{3}{2} \sigma' H^i (\epsilon \sigma_3)^{ij} - m_\Psi H^i \epsilon^{ij} \right] \eta^j \,,
\end{aligned} \qquad (44)
$$

where η^i satisfies the Killing equation (33) and the condition (34), requires that the five-dimensional masses of the scalars and fermion satisfy

$$
\begin{aligned}
m_{H^{1,2}}^2 &= (c^2 \pm c - \frac{15}{4}) k^2 + \left(\frac{3}{2} \mp c \right) \sigma'' \,, \\
m_\Psi &= c \sigma' \,,
\end{aligned} \qquad (45)
$$

where c remains an arbitrary dimensionless parameter. Using Eq. (8) we can identify

$$
a = c^2 \pm c - \frac{15}{4} \quad \text{and} \quad b = \frac{3}{2} \mp c \,. \qquad (46)
$$

Thus, we find that $\alpha = |c + 1/2|$ for H^1 and Ψ_L, and $\alpha = |c - 1/2|$ for H^2 and Ψ_R. Assuming that H^1 and Ψ_L are even, while H^2 and Ψ_R are odd, then the Kaluza-Klein masses (identical for both the even and odd modes) are determined by the equation

$$
\frac{J_{|c+1/2|}\left(\frac{m_n}{k}\right)}{Y_{|c+1/2|}\left(\frac{m_n}{k}\right)} = \frac{J_{|c+1/2|}\left(\frac{m_n}{k} e^{\pi kR}\right)}{Y_{|c+1/2|}\left(\frac{m_n}{k} e^{\pi kR}\right)} \,. \qquad (47)
$$

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In fact using Eq.(22) for the even modes, and Eq.(27) for the odd modes, we can again see that the odd and even Kaluza-Klein modes, $n = 1, 2, \ldots$ have identical masses which are approximately given by

$$m_n \simeq (n + \frac{c}{2} - \frac{1}{2})\pi k e^{-k\pi R} \ . \tag{48}$$

The massless modes of H^1 and Ψ_L are given by

$$
\begin{aligned}
H^1(x, y) &= \frac{e^{(3/2-c)\sigma}}{\sqrt{2\pi R N_0}} H^{1\,(0)}(x) + \ldots , \\
\Psi_L(x, y) &= \frac{e^{(2-c)\sigma}}{\sqrt{2\pi R N_0}} \Psi_L^{(0)}(x) + \ldots ,
\end{aligned}
\tag{49}
$$

where

$$N_0^2 = \frac{e^{2\pi k R(1/2-c)} - 1}{2\pi k R(1/2 - c)} \ . \tag{50}$$

When $c = 1/2$, we have $N_0 = 1$ which signals the conformal limit, where the kinetic terms are again independent of the y coordinate. Thus, as explained in the vector multiplet case, the massless modes are not localized by the AdS space and will couple to the two boundaries with equal strength. For values of $c < 1/2$, the AdS space will localize the massless modes towards the boundary $y^* = \pi R$. As c becomes more negative, the localization at the $y^* = \pi R$ boundary becomes increasingly more effective. On the other hand when $c > 1/2$, the massless modes will be localized towards the $y^* = 0$ boundary. In this case the localization becomes more effective the larger the value of c.

The odd fields do not have a massless mode, since this is not consistent with the orbifold condition. Therefore, as in the case of the vector multiplet, the massless sector of the hypermultiplet, (from H^1 and Ψ_L), forms an $N = 1$ supersymmetric chiral multiplet.

4   Phenomenological implications

In the original proposal in Ref. [2], the SM fields were assumed to be confined on the TeV-brane. The question naturally arises then of whether the SM fields can actually live in the bulk. We will first consider models with bulk fields but without supersymmetry. A non-supersymmetric model can be constructed, but eventually we will see that many more interesting possibilities exist for supersymmetric models.

4.1   The Standard Model in the bulk

We have seen that the massless modes of a 5D scalar or fermion field in the AdS slice can be localized at either of the two boundaries, depending on the value of the dimensionless mass

parameter c. This continues to be true for massless fermions, even without supersymmetry, due to the fact that chiral symmetry protects the massless mode. However, without supersymmetry, the massless mode of the bulk scalar field will receive radiative corrections of order the Planck scale. Thus, it would appear that the hierarchy problem is again reintroduced, as in flat space. This dire conclusion is not in fact realised, because the nonzero Kaluza-Klein scalar modes are localized near the TeV-brane and consequently have TeV scale masses. Thus, even though the scalar zero mode becomes heavy, the mass of the lightest nonzero Kaluza-Klein mode is still of order the TeV scale, and therefore the Higgs can be associated with this lightest Kaluza-Klein state. This is effectively like having the Higgs field on the TeV-brane.

If the fermion fields are located in the bulk, then the gauge bosons must necessarily reside in the bulk. In this case, the Kaluza-Klein excitations of the gauge bosons can induce four-fermion interactions that are severely constrained by the experimental data [20, 21, 22]. To study these constraints in our case, let us consider the five-dimensional gauge coupling between the bulk gauge boson and fermion. It is given by

$$\int d^4x \int dy \sqrt{-g}\, g_5 \bar{\Psi}(x,y) i\gamma^\mu A_\mu(x,y) \Psi(x,y) \;, \tag{51}$$

where g_5 is the five-dimensional gauge coupling. Using the expression for the zero-mode fermion (49), the gauge coupling of a gauge boson Kaluza-Klein mode n to the zero-mode fermions is

$$g^{(n)} = g \left(\frac{1-2c}{e^{(1-2c)\pi kR} - 1} \right) \frac{k}{N_n} \int_0^{\pi R} dy\, e^{(1-2c)\sigma} \left[J_1(\frac{m_n}{k}e^\sigma) + b_1(m_n)Y_1(\frac{m_n}{k}e^\sigma) \right] \;, \tag{52}$$

where $g = g_5/\sqrt{2\pi R}$ is the four-dimensional gauge coupling, and c is defined in Eq. (8). In Fig. 1, we show the ratio $g^{(n)}/g$, for $n = 1, 2, 3$, as a function of c. When c is large and negative,
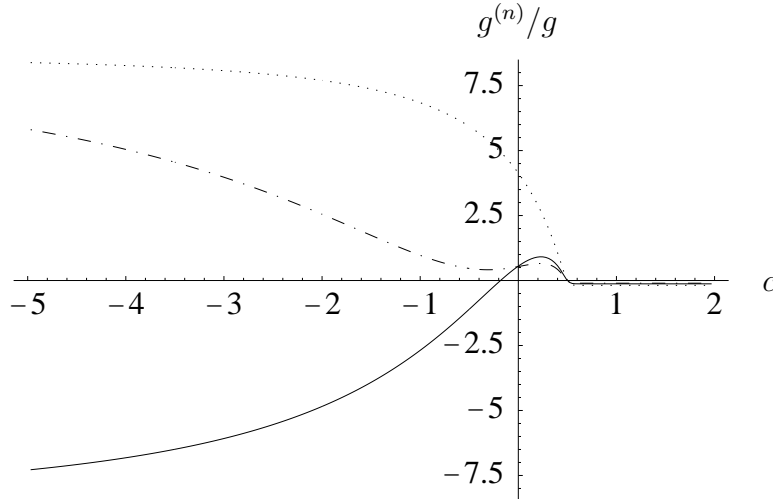

Figure 1: *The ratio of the gauge couplings, $g^{(n)}/g$, for $n = 1$ (dotted line), $n = 2$ (solid line) and $n = 3$ (dashed-dotted line), as a function of the dimensionless fermion mass parameter c.*

the fermion is localized near the TeV-brane and the ratio $g^{(1)}/g$ approaches the asymptotic

limit $g^{(1)}/g \simeq \sqrt{2\pi kR} \simeq 8.4$, which corresponds to a TeV-brane fermion [5, 6]. In this region the first excited Kaluza-Klein mode of the gauge boson couples strongly to the fermion zero-mode, leading to a restrictive lower bound on the first excited Kaluza-Klein mass scale $m_1 \gtrsim 20$ TeV [5]. At $c = 0$, we recover the massless fermion case considered in Ref. [8]. As the value c of the bulk fermion mass increases, we find that the bound considerably weakens, because the fermion zero mode couples less strongly. At the conformal limit $c = 1/2$, the coupling completely vanishes due to the fact that the five-momentum is conserved in this limit. For $c > 1/2$, the coupling quickly becomes universal for all fermion masses. This is because the fermions are now localized near the Planck-brane, where the wavefunction of the Kaluza-Klein gauge bosons is constant. In this region the asymptotic limit $g^{(1)}/g \simeq -0.2$, agrees with the case of a Planck-brane fermion [6]. If all fermions have equal five-dimensional masses, the bound from electroweak precision experiments on the first excited Kaluza-Klein mass is

$$m_1 \gtrsim 2.1 \left(\frac{g^{(1)}}{g} \right) \text{ TeV} , \tag{53}$$

where $m_1 \simeq 2.5 k e^{-\pi kR}$. Thus we see that in the region where $c \gtrsim 1/2$, the lower bound is fairly innocuous. The coupling of the higher-order Kaluza-Klein modes ($n > 1$) is always smaller than the coupling of the first excited state, and thus gives negligible contributions to any four-fermion operator (unless the fermion is on the TeV-brane). As can be seen in Fig. 1, the ratio of the couplings, $g^{(2)}/g$, and $g^{(3)}/g$, continue to remain c-independent for $c \gtrsim 1/2$.

4.2 Fermion mass hierarchy

In the original scenario [2], the warp factor $e^{-\sigma}$, was used to solve the gauge hierarchy problem by generating the TeV scale from the Planck scale. The fermion mass hierarchy problem was not addressed there because the fermions were confined to the brane and the Yukawa interactions were conformally invariant. However, if we allow the fermions to live in the bulk, a similar use of the warp factor can be used to explain the fermion mass hierarchies. For neutrino masses this idea has been considered in Ref. [7]. Here we will analyze it for the quarks and leptons. Our setup is as follows. For each fermion flavour i, we have two five-dimensional Dirac fermions $\Psi_{iL}(x,y)$, and $\Psi_{iR}(x,y)$, while for simplicity the Higgs $H(x)$ will be localized on the TeV-brane. Thus, the five-dimensional action will have the Yukawa coupling term

$$\int d^4x \int dy \sqrt{-g} \, \lambda_{ij}^{(5)} H(x) \Big(\bar{\Psi}_{iL}(x,y) \Psi_{jR}(x,y) + h.c. \Big) \delta(y - \pi R)$$
$$\equiv \int d^4x \, \lambda_{ij} H(x) \Big(\bar{\Psi}_{iL}^{(0)}(x) \Psi_{jR}^{(0)}(x) + h.c. + \dots \Big) , \tag{54}$$

where $\lambda_{ij}^{(5)}$ are the five-dimensional Yukawa couplings and λ_{ij} define the effective four-dimensional Yukawa couplings of the zero modes, $\Psi_{iL}^{(0)}$ and $\Psi_{jR}^{(0)}$. If each fermion field has a bulk mass term parametrized by c_{iL} and c_{jR}, then the fermion zero modes will develop an exponential profile

which gives rise to the following four-dimensional Yukawa couplings

$$\lambda_{ij} = \frac{\lambda_{ij}^{(5)}k}{N_{iL}N_{jR}}e^{(1-c_{iL}-c_{jR})\pi kR} \ , \tag{55}$$

where

$$\frac{1}{N_{iL}^2} \equiv \frac{1/2 - c_{iL}}{e^{(1-2c_{iL})\pi kR} - 1} \ , \tag{56}$$

and similarly for N_{jR}. Note that in deriving (55), the Higgs must be rescaled by an amount $H(x) \to e^{\pi kR}H(x)$, in order to obtain a canonically normalized kinetic term. We see then from Eq. (55), that exponentially-small Yukawa couplings can be generated for values of c_{iL} and c_{jR} slightly larger than $1/2$.

The precise flavour structure of the Yukawa coupling matrix, λ_{ij}, will depend on the values of c_{iL} and c_{jR}. We can consider two simple limits, which in some sense represents the two possible extremes for the flavour structure. Suppose first that we have a left-right symmetric theory such that $c_{iL} = c_{iR}$. In Fig. 2 we have plotted the diagonal element of the Yukawa matrix, Eq. (55), in this left-right symmetric limit. We have assumed, for simplicity, that $\lambda_{ij}^{(5)}k \sim 1$. For $c_{iL} > 1/2$, we clearly see in Fig. 2 the exponential damping of Eq. (55), where the diagonal elements of Eq. (55) simplifies to (for $kR \gg 1$)

$$\lambda_{ii} \simeq \lambda_{ii}^{(5)}k\left(c_{iL} - 1/2\right)e^{-2(c_{iL}-1/2)\pi kR} \ . \tag{57}$$

An electron Yukawa coupling $\lambda_e \sim 10^{-6}$ can be generated for $c_{eL} \simeq 0.64$ (assuming $kR \sim 12.46$). On the other hand, a top Yukawa coupling $\lambda_t \sim 1$ can be obtained for $c_{tL} = -1/2$, since the exponential factor in (55), disappears for $c_{iL} < 1/2$.

Alternatively, we can suppose that the right-handed fermions all have $c_{jR} = 1/2$. Now, the effective Yukawa couplings, in the limit that $kR \gg 1$ and $c_{iL} > 1/2$ become

$$\lambda_{ii} \simeq \frac{\lambda_{ii}^{(5)}k}{\sqrt{2\pi kR}}\sqrt{c_{iL} - 1/2}\,e^{-(c_{iL}-1/2)\pi kR} \ . \tag{58}$$

In this case the exponential factor is more dominant compared to the left-right symmetric assumption. This can clearly be seen in Fig. 2.

It is easy to understand the behaviour of the curves in Fig. 2. For $c_{iL} > 1/2$, the Yukawa couplings quickly become exponentially-small since fermions are localized near the Planck-brane and have an exponentially small overlap with the Higgs, which is located on the TeV-brane. In this way we see that the mass of the lightest fermion states of the SM can be naturally generated for values of $c_{iL} > 1/2$. The heavier fermions (c, τ, b) require a larger overlap with the Higgs and cannot be confined near the Planck-brane. In this case the values of the mass parameters must be close to the conformal limit $(c_{iL} \simeq 1/2)$, where fermions are not localized on either of the two branes. The top quark having a comparable mass to the VEV of the Higgs, needs the largest overlap of all, and must be localized on the TeV-brane $(c_{tL} \lesssim -1/2)$. Again
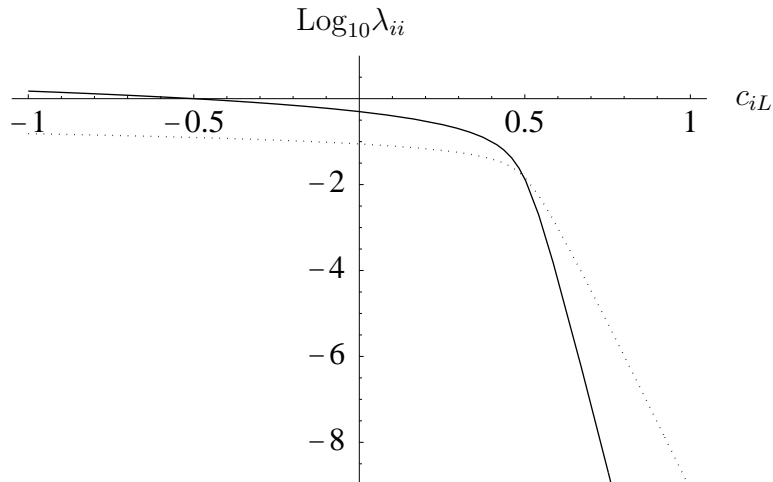
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Figure 2: *The effective four-dimensional Yukawa coupling λ_{ii} as a function of c_{iL}, for the case $c_{iL} = c_{iR}$ (solid line) and $c_{iR} = 1/2$ (dotted line) .*

we must stress that this scenario does not predict the fermion mass spectrum (which would correspond to predicting the values of c_{iL} and c_{jR}). However, it does offer a possible explanation for the mass hierarchy between fermion families. This idea is similar to Ref. [23], where the exponential overlap between the Higgs and fermion wavefunctions also generated fermion mass hierarchies.

It is important to realise, however, that if the fermions are localized at different points (by assuming that they have different 5D masses c_i), then they will couple differently to the Kaluza-Klein gauge bosons. This can give rise to large flavour-changing neutral current (FCNC) processes [22]. For example, in the model of Ref. [23], a lower bound on the compactification scale of $25 - 300$ TeV is obtained [22] (the range in the bound depends on whether or not the induced FCNC processes violate the CP symmetry). This would also seem to be a problem for our scenario. However, in our scenario the FCNC constraints lead to a much smaller lower bound on the first excited Kaluza-Klein mass m_1. For the case represented by Eq.(57) we have $m_1 \gtrsim 2 - 30$ TeV, while $m_1 \gtrsim 0.2 - 2.5$ TeV in the case of Eq.(58). The lower bounds are much reduced because, as shown in Fig. 1, the Kaluza-Klein gauge-bosons couple almost universally to any fermion with $c_{iL} \gtrsim 1/2$. This is the case for the first and second family. The third-family fermions do not have a universal coupling, since $c_{iL} < 1/2$, but in this case the FCNC constraints are much weaker [22].

4.3   Higher-dimensional operators

When fermions are confined on the TeV-brane, as in the original setup [2], higher-dimensional operators are suppressed by the TeV scale and not the Planck-scale [10]. In the absence of any discrete symmetries, this of course leads to proton decay problems. In addition, constraints on $K - \bar{K}$ mixing requires that the dimension-six operator, $(d\bar{s})^2/M_4^2$ needs to be suppressed by

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at least an effective four-dimensional mass scale $M_4 \sim 1000$ TeV. There is also the problem of generating large neutrino masses.

If the fermion fields are instead in the bulk, then we can ask whether effective higher-dimensional operators are suppressed. As we saw in Section 3, each fermion zero mode has a wavefunction proportional to $e^{(2-c_i)\sigma}$, and depending on the values of c_i can lead to an exponential suppression of mass scales. Let us consider first the following generic four-fermion operators which are relevant for proton decay and $K - \bar{K}$ mixing:

$$\int d^4x \int dy \ \sqrt{-g} \frac{1}{M_5^3} \bar{\Psi}_i \Psi_j \bar{\Psi}_k \Psi_l \equiv \int d^4x \ \frac{1}{M_4^2} \bar{\Psi}_i^{(0)} \Psi_j^{(0)} \bar{\Psi}_k^{(0)} \Psi_l^{(0)} \ , \tag{59}$$

where the effective four-dimensional mass scale M_4 is

$$\frac{1}{M_4^2} = \frac{2k}{M_5^3} \frac{1}{N_i N_j N_k N_l} \frac{e^{(4-c_i-c_j-c_k-c_l)\pi kR} - 1}{(4 - c_i - c_j - c_k - c_l)} \ , \tag{60}$$

and N_i are defined in Eq. (56). For the values $1/2 \lesssim c_i \lesssim 1$, the four-dimensional suppression scale, M_4 ranges from the TeV scale (for $c_i \simeq 1/2$) to M_P (for $c_i \simeq 1$). Therefore, we can only obtain the necessary suppression for proton decay if the mass parameters satisfy $c_i \gtrsim 1$. This, of course, means that the light fermions are localized on the Planck-brane. Unfortunately, for $c_i \gtrsim 1$ fermion masses are much too small (see Eq. (55)). This is because (in non-supersymmetric models), the Higgs field must live on the TeV-brane and will have a very small overlap with the fermions. Thus, it is impossible to suppress proton decay operators and have fermion masses. On the other hand, we find that the constraints on $K - \bar{K}$ mixing, that require $M_4 \gtrsim 1000$ TeV, can be satisfied for values of c_i compatible with generating fermion masses.

Finally, let us consider the dimension-five operator responsible for generating Majorana neutrino masses. Again assuming that the left-handed fields are in the bulk we obtain

$$\int d^4x \int dy \ \sqrt{-g} \frac{1}{M_5^2} \Psi_{iL}^T C_5 \Psi_{iL} H(x) H(x) \delta(y - \pi R) \equiv \int d^4x \ \frac{1}{M_4} \Psi_{iL}^{(0)\,T} C_5 \Psi_{iL}^{(0)} HH \ , \tag{61}$$

where C_5 is the charge-conjugation operator. For $c_{iL} > 1/2$, M_4 is given by

$$M_4 \simeq \frac{M_5^2}{k} e^{-2(1-c_{iL})\pi kR} \ . \tag{62}$$

Using the relation (58), we can relate the neutrino masses to the fermion masses:

$$m_{\nu_i} \simeq 10^2 \ \lambda_{ii}^2 \ \text{GeV} \ , \tag{63}$$

and leads to neutrino mass values of $(1, 10^3, 10^7)$ eV for $(\nu_e, \nu_\mu, \nu_\tau)$. While these neutrino masses are not ruled out by collider experiments, they can be problematic for cosmology. In particular, if the neutrinos are stable, then the masses do not satisfy the bound from overclosing the universe. This constraint is easily avoided if the neutrinos decay, but this now requires a more detailed analysis.

In conclusion, the requirement that the Higgs field is localized on the TeV-brane, in order to solve the gauge-hierarchy problem, is in direct conflict with proton decay, although FCNC constraints can be safely avoided if fermions live in the bulk. To delocalize the Higgs from the TeV-brane and allow new scenarios we need supersymmetry.

4.4 Supersymmetric models

In the case of non-supersymmetric theories we were forced to have the Higgs field localized on the TeV-brane, with a small overlapping on the Planck-brane. However, if supersymmetry is realized, we can relax the above assumption. The Higgs fields can now live anywhere in the bulk since their masslessness will be stable from radiative corrections. We will consider two new possibilities. The Higgs can be either

a) not localized at all, $c = 1/2$ in Eq. (49) (i.e. conformally flat limit), or

a) localized on the Planck-brane, $c \gg 1/2$ in Eq. (49).

These two possibilities, (a) and (b), allow the fermions (and their superpartners) to be placed on the Planck-brane. The main motivation for having the SM fermion living on the Planck-brane is to avoid dangerous higher-dimensional operators that, as discussed in the previous section, could induce large proton decay. By placing the fermions on the Planck-brane, any higher-dimensional operator will be suppressed by M_P [6] (or the slightly smaller GUT scale of 10^{16} GeV). This occurs if either the SM fermions are actually confined to the Planck-brane as four-dimensional fields, or they are bulk fields with five-dimensional masses $c \gg 1$ (see discussion below Eq. (60)). In the latter case, the fermions form part of the five-dimensional hypermultiplets that, as shown in section 3, are localized by the AdS metric on the Planck-brane. Unfortunately with the Higgs off the TeV-brane, the Yukawa couplings are no longer exponentially suppressed, and the fermion mass hierarchy cannot be explained by the mechanism outlined in Section 4.2.

Finally, we will consider the SM gauge-boson as a five-dimensional bulk field. This is a necessary consequence if some of the SM fermions (or the Higgs) live in the bulk. There is also another motivation here. If the SM gauge-bosons live in the bulk, they will behave as messenger fields between the two branes, communicating any physics on the TeV-brane to the Planck-brane. In particular, if supersymmetry is assumed to be broken on the TeV-brane, they will communicate it to the Planck-brane. But also Planckian (stringy) physics can be communicated to the Planck-brane. Note that at the TeV-brane, Planckian physics is rescaled down to TeV energies. Therefore a gauge boson can be produced at the Planck-brane with a TeV energy and then propagate to the TeV-brane. At the TeV-brane, it will interact with Planck scale excitations of the gauge boson, which now have TeV masses. Thus, we expect that in this scenario we will be able to test GUT or theories of quantum gravity at the TeV.

4.4.1 Supersymmetry breaking on the TeV-brane

Let us assume that supersymmetry is broken on the brane at $y^* = \pi R$. Since all mass scales at $y^* = \pi R$, are rescaled to $M_P e^{-\pi k R}$, one can generate small supersymmetry-breaking soft masses. This presents a new alternative to models with gaugino condensation where the small scale of supersymmetry breaking is generated by dimensional transmutation.

Since we assume that the SM gauge superfields live in the 5D bulk, they couple to the TeV-brane and can feel the breaking of supersymmetry at tree-level. In particular, gaugino mass

terms can be generated from the TeV-brane spoiling the supersymmetric condition Eq. (37). This will modify the Kaluza-Klein decomposition of the gaugino fields given in section 3. For example, the full Kaluza-Klein mass spectrum will be shifted up with respect to the boson sector. There will no longer be a massless gaugino mode in the spectrum. Although the precise value of the Kaluza-Klein masses and eigenfunctions will be left for future work, we can infer here some of the consequences.

The breaking of supersymmetry on the $y^* = \pi R$ brane will be communicated to the rest of SM particle superfields by the gauge superfields. Squarks and sleptons on the Planck-brane will receive masses at the quantum level. Since gauge interactions are flavour-independent, the scalar masses will be flavour diagonal, solving the supersymmetric flavour problem. Notice that not only the massless mode but all the Kaluza-Klein tower will mediate the breaking of supersymmetry similarly as in Ref. [11]. Therefore we expect that the sparticle spectrum will be very similar to that in theories of supersymmetry-breaking by TeV compactifications [11]. In particular we expect a mass gap between the scalar masses and the gaugino masses. This is because the scalars will receive masses at the one-loop level

$$m_i^2 \sim \frac{\alpha_i}{4\pi}(\text{TeV})^2 \ , \tag{64}$$

where α_i is the gauge coupling of the gauge group under which the scalar i transforms. The scalar masses will be an order of magnitude smaller than the gaugino masses and fulfil the relation

$$\frac{m_i^2}{m_j^2} = \frac{\alpha_i}{\alpha_j} \ . \tag{65}$$

The right-handed slepton will be the lightest scalar. However, the lightest supersymmetric particle (LSP) will be the gravitino. Since it couples to the TeV-brane by $1/M_P$-suppressed interactions, its mass will be of order TeV^2/M_P. This represents a very light gravitino with mass $m_{3/2} \sim 10^{-4}$ eV and satisfies the usual constraints from cosmology and collider experiments [24].

What about the Higgs? In model (a) the Higgs will also couple to the TeV-brane and Higgsino masses can be induced at tree-level. In general, the scalar Higgs will also get masses at tree-level. This can be problematic since the electroweak scale will be of order TeV [4]. Therefore we must require that only the Higgsino mass is induced at the tree-level. The origin of such a pattern of supersymmetry-breaking masses must be addressed by the fundamental (string) theory. Of course, even if the Higgs soft masses are zero at tree-level, they will be induced at the loop level. As in the case of the squark and slepton masses, we expect that the soft Higgs masses will be positive at one-loop level. Nevertheless, for the Higgs coupled to the top, there are also negative two-loop effects coming from the top/stop that can dominate the one-loop contribution and make the Higgs mass negative [11]. This can lead to electroweak breaking at a scale an order of magnitude smaller than the supersymmetry-breaking scale. For

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[4]We could lower the supersymmetry breaking scale from a TeV to 100 GeV, but in this case the squark and slepton masses, that arise at the loop level, will be too small. Another alternative would be to reduce the couplings of the Higgs to the TeV-brane by increasing its 5D mass, $c \gtrsim 1/2$, and having the Higgs slightly localized towards the Planck-brane.

model (b), however, the Higgsino mass is zero since the Higgs superfields do not couple to the TeV-brane. This gives rise to the usual μ-problem. The simple solution that we envisage is to introduce a singlet field that couples to the Higgsinos. This singlet can get a VEV at the electroweak scale and induce Higgsino masses.

Finally, we want to comment on a different alternative to the models considered here, where only gravity lives in the five-dimensional bulk, while the complete SM, including the gauge fields, lives on the Planck-brane. Supersymmetry breaking arising from the brane at $y^* = \pi R$, would then be communicated by gravity to the SM fields. In this case, soft masses will be of order $(M_P e^{-\pi k R})^2 / M_P$, and implies that the radius of the extra dimension satisfies $kR \simeq 6$. This model is very similar to gravity-mediated models of supersymmetry-breaking and we do not expect any new interesting phenomenology.

5   Conclusion

Bulk fields living in a slice of AdS have different behaviour, depending on their spin. We have studied the Kaluza-Klein mass spectrum and wavefunctions of scalars, gauge bosons and fermions and analyzed the conditions for supersymmetry. In the massless sector we find that the scalar and fermion fields can be localized by the AdS geometry on either brane (depending on their 5D masses), while the gauge bosons are always nonlocalized. Furthermore, the gauge bosons couple to each brane with equal strength, and therefore provide a window for physics of the distant brane.

We have also studied the phenomenological implications of having the SM fields propagating in a slice of AdS. In non-supersymmetric theories, only the Higgs is required to be localized on the TeV-brane. The precise scale of the TeV-brane is constrained by electroweak precision data, and strongly depends on the 5D fermion masses as shown in Eq. (53) and Fig. 1. Remarkably, there is no significant bound if the 5D fermion masses satisfy $m_\Psi > \sigma'/2$. For these values of m_Ψ, the four-dimensional massless fermions are slightly localized near the Planck-brane and have an exponentially small coupling to the Higgs. We have shown that this could explain the fermion mass hierarchy by confining the light fermions to the Planck-brane, and the heavy fermions to the TeV-brane. Similarly, by studying higher-dimensional operators, we have shown that constraints from proton decay forces the fermions to be strongly confined towards the Planck-brane. Unfortunately, these constraints forbid the generation of fermion masses, because now there is very little wavefunction overlap with the Higgs.

If supersymmetry is present, the massless Higgs mode is protected from radiative corrections and the Higgs can now live off the TeV-brane. Thus, higher-dimensional proton decay operators can be suppressed, while at the same time having fermion masses. However, the warp factor needed to explain the hierarchy becomes impotent, since the Higgs and fermions are located near the Planck-brane. Assuming that the source of supersymmetry-breaking is on the TeV-brane, the scale of supersymmetry-breaking will be of $\mathcal{O}(\text{TeV})$. This provides a new alternative to gaugino condensation models. If the gauge fields live in the bulk, gaugino mass terms will be

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generated at tree-level. The gauge fields will communicate the supersymmetry breaking to the Planck-brane where the squarks and sleptons live. Thus, the supersymmetric flavour problem is naturally solved. This model leads to a supersymmetric mass spectrum where the right-handed slepton is the lightest scalar and the LSP is a very light gravitino. One particularly favourable model, is when the Higgs field is conformally flat ($c = 1/2$) and nonlocalized. In this case the Higgsino mass could also be induced at tree-level.

Finally, we reiterate that the whole scenario, including the details of the source branes, must eventually be embedded into some underlying theory (such as string theory). In particular, such a fundamental theory must provide a tree-level supersymmetry-breaking mechanism on the TeV-brane. It is appealing that the hierarchies in the fundamental scales of physics can be directly related to the geometry of space-time. This fact alone, warrants further investigation of these scenarios.

Acknowledgements

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